GRAVITATION

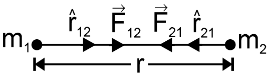
GRAVITATION : **Universal Law of Gravitation**

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = G\frac{m_1 m_2}{r^2}$$

where $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$ is the universal gravitational constant.

Newton's Law of Gravitation in vector form :

$$\vec{F}_{12} = \frac{Gm_1m_2}{r^2}\ \hat{r}_{12} \quad \& \quad \vec{F}_{21} = \frac{Gm_1m_2}{r^2}$$



Now $\hat{r}_{12} = -\hat{r}_{21}$, Thus $\vec{F}_{21} = \dfrac{-G\, m_1\, m_2}{r^2}\ \hat{r}_{12}$.

Comparing above, we get $\vec{F}_{12} = -\vec{F}_{21}$

Gravitational Field $E = \dfrac{F}{m} = \dfrac{GM}{r^2}$

Gravitational potential : gravitational potential,

$$V = -\frac{GM}{r}. \qquad E = -\frac{dV}{dr}.$$

1. **Ring.** $V = \dfrac{-GM}{x \text{ or } (a^2 + r^2)^{1/2}}$ & $E = \dfrac{-GMr}{(a^2 + r^2)^{3/2}}\hat{r}$

or $E = -\dfrac{GM\cos\theta}{x^2}$

Gravitational field is maximum at a distance,

$r = \pm\ a/\sqrt{2}$ and it is $-\ 2GM/3\sqrt{3}\,a^2$

2. Thin Circular Disc.

$$V = \frac{-2GM}{a^2}\left[\left[a^2+r^2\right]^{\frac{1}{2}} - r\right] \quad \& \quad E = -\frac{2GM}{a^2}\left[1 - \frac{r}{\left[r^2+a^2\right]^{\frac{1}{2}}}\right] = -\frac{2GM}{a^2}\left[1-\cos\theta\right]$$

3. Non conducting solid sphere

(a) Point P inside the sphere. $r \le a$, then

$$V = -\frac{GM}{2a^3}(3a^2 - r^2) \ \& \ E = -\frac{GMr}{a^3}, \text{ and at the centre } V = -\frac{3GM}{2a} \text{ and } E = 0$$

(b) Point P outside the sphere .

$r \ge a$, then $V = -\dfrac{GM}{r}$ & $E = -\dfrac{GM}{r^2}$

4. Uniform Thin Spherical Shell / Conducting solid sphere

(a) Point P Inside the shell.

$r \le a$, then $V = \dfrac{-GM}{a}$ & $E = 0$

(b) Point P outside shell.

$r \ge a$, then $V = \dfrac{-GM}{r}$ & $E = -\dfrac{GM}{r^2}$

VARIATION OF ACCELERATION DUE TO GRAVITY :

1. Effect of Altitude

$$g_h = \frac{GM_e}{(R_e+h)^2} = g\left(1+\frac{h}{R_e}\right)^{-2} \simeq g\left(1-\frac{2h}{R_e}\right) \text{ when } h << R.$$

2. Effect of depth $\quad g_d = g\left(1-\dfrac{d}{R_e}\right)$

3. Effect of the surface of Earth

The equatorial radius is about 21 km longer than its polar radius.

We know, $g = \dfrac{GM_e}{R_e^2}$ Hence $g_{pole} > g_{equator}$.

SATELLITE VELOCITY (OR ORBITAL VELOCITY)

$$v_0 = \left[\frac{GM_e}{(R_e+h)}\right]^{\frac{1}{2}} = \left[\frac{gR_e^2}{(R_e+h)}\right]^{\frac{1}{2}}$$

When h << R_e then $v_0 = \sqrt{gR_e}$

$\therefore \quad v_0 = \sqrt{9.8 \times 6.4 \times 10^6}$ = 7.92 × 10^3 ms^{-1} = 7.92 km s^1

Time period of Satellite

$$T = \frac{2\pi(R_e + h)}{\left[\frac{gR_e^2}{(R_e+h)}\right]^{\frac{1}{2}}} = \frac{2\pi}{R_e}\left[\frac{(R_e+h)^3}{g}\right]^{\frac{1}{2}}$$

Energy of a Satellite

$U = \frac{-GM_e m}{r}$ K.E. $= \frac{GM_e m}{2r}$; then total energy $\rightarrow$ $E = -\frac{GM_e m}{2R_e}$

Kepler's Laws

Law of area :

The line joining the sun and a planet sweeps out equal areas in equal intervals of time.

Areal velocity $= \frac{\text{area swept}}{\text{time}} = \frac{\frac{1}{2}r(rd\theta)}{dt} =7 \frac{1}{2} r^2 \frac{d\theta}{dt}$ = constant .

Hence $\frac{1}{2} r^2 \omega$ = constant. **Law of periods :** $\frac{T^2}{R^3}$ = constant